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**Project 4 – Degradation of Data Integrity**

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Course number: CST - 305

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**Project 4 – Degradation of Data Integrity**

**PART 1: Data Integrity with 3 x 3 Matrix**

**Description**

This assignment has two parts: theoretical and practical. You will build a system of ODE that models the loss or corruption of data in a digital storage system, as follows:

*x ′ 1 = a11x1 + · · · + a1nxn + f1*

*x ′ 2 = a21x1 + · · · + a2nxn + f2*

*. . . . . . . . . . . . . . . . . . . . . . . .*

*x ′ m = am1x1 + · · · + amnxn + fm*

Assume that the functions *aij(t)* and *fj(t)* are given, while the functions *x1(t), …, xn(t)* are the unknowns. Remember that the rate of change is given by:

*rate of change = input rate – output rat*e.

Consider the three processors shown in the figure. Let us suppose, for the sake of discussion that processor 𝐴 contains 100 Mbytes of data in which 25 Mbytes is used for I/O and 75 Mbytes for internal housekeeping. Suppose processor 𝐵 and 𝐶 also contains 100 Mbytes of data (75 MB for housekeeping and 25 MB for I/O). Data is pumped into and out of the processors as indicated in the figure. The I/O plus the data exchanged between the three processors and the I/O data propagated to processor C and going downstream to the network are well processed.

Diagram

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1. Construct a mathematical model that describe the number of I/O + data 𝑥1 (𝑡),𝑥2 (𝑡) 𝑎𝑛𝑑 𝑥3 (𝑡) of network data in processors 𝐴,𝐵 𝑎𝑛𝑑 𝐶, respectively, at time 𝑡.
2. Put the mathematical model (which is a system of 1st order differential equations) into the form: 𝑥̇=𝐴(𝑡)𝑥(𝑡)+𝑓(𝑡),𝑤𝑖𝑡ℎ 𝑓(𝑡)=0.

Apply the equation, then we get

1. Find the eigenvalues of matrix 𝐴.

By using Cayley-Hamilton Theorem, we can evaluate eigenvalues of matrix A.

Cayley-Hamilton Theorem: = 0

= 0

When we calculate the eigenvalues with the eigenvalue calculator from WolframAlpha.

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Once we evaluate the characteristic polynomial equation within the matrix, we get

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When we evaluate the roots for the equation above, then we get

**PART 2: Data Integrity with 2 x 2 Matrix**

Two Processors 𝐴 and 𝐵 have each 100 Mbytes of memory. Initially, 100 Mbytes of data + I/O is stored in processor 𝐴 and 100 Mbytes of data + I/O is stored in processor 𝐵. The system is closed in that data + I/O is pumped only between the processors, as shown in the Figure.

Diagram

Description automatically generated

1. Use the information given in the figure to construct a mathematical model for the number of I/O data 𝑥1(𝑡) 𝑎𝑛𝑑 𝑥2(𝑡) at time t in processors 𝐴 and 𝐵, respectively.
2. Write the system of linear differential equations in mathematical form 𝑥̇=𝐴(𝑡)𝑥(𝑡)+𝑓(𝑡),𝑤𝑖𝑡ℎ 𝑓(𝑡)=0.

Apply the equation then we get,

1. Calculate from matrix 𝐴 found in the system of 1st order linear differential equations,

The computation of can be written as . This infinite series can be reduced to a polynomial in t. Thus,

Where are functions of t which must be determined for each A.

Since n = 2 for this problem, we can say

When we evaluate the function, then we get

Let A be a matrix having n rows and n columns, and define

Then, if is an eigenvalue of At,

Therefore,

Since n = 2, we have

We need to find to evaluate the , we use Cayley-Hamilton Theorem to evaluate.

Substitute to the theorem above, we get

Since , we get

Solve for

Apply and to again, then we get

1. Find the solution to the initial value problem 𝑥(0)=1,𝑥̇(0)=−1 by matrix method.

To solve IVP, we have given 𝑥(0) = 1, 𝑥̇(0) = −1. Any IVP in which the differential equations are all linear with constant coefficients, can be reduced to the matrix system.

Where A is a matrix of constants. The solution to the above equation is

However, we have a IVP for homogeneous where f(t) = 0, then the solution is

Since we have found , we have to evaluate and c to find the solution.

As mentioned above, we have given 𝑥(0) = 1, 𝑥̇(0) = −1 where then, when we evaluate,

Apply and c to the solution, then

\* Solution

1. Choose the appropriate software packages to generate graphs for the solution in (d) or animate the data, as best suited for your project.

To visualize the ODE solution, I used python. Python is a programming language that is generally used for data science.

First, import the packages to visualize and calculate the equations.

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After that, we define the function and program for the visualization.

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Lastly, visualize the solution.

Chart, line chart

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